Modified $f(G)$ gravity models with curvature-matter coupling

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A modified f(G) gravity model with coupling between matter and geometry is proposed, which is described by the product of the Lagrange density of the matter and an arbitrary function of the Gauss-Bonnet term. The field equations and the equations of motion corresponding to this model show the non-conservation of the energy-momentum tensor, the presence of an extra-force acting on test particles and the non-geodesic motion. Moreover, the energy conditions and the stability criterion at de Sitter point in the modified f(G) gravity models with curvature-matter coupling are derived, which can degenerate to the well-known energy conditions in general relativity. Furthermore, in order to get some insight on the meaning of these energy conditions, we apply them to the specific models of f(G) gravity and the corresponding constraints on the models are given. In addition, the conditions and the candidate for late-time cosmic accelerated expansion in the modified f(G) gravity are studied by means of conditions of power-law expansion and the equation of state of matter less than $-\frac{1}{3}$.

PACS numbers: 98.80.-k

Keywords: f(G) gravity; Energy conditions; Acceleration

I. INTRODUCTION

According to recent observational data sets[1, 2], our current universe is flat and undergoing a phase of the accelerated expansion which started about five billion years ago. In principle, this phenomenon can be explained by either dark energy (see, for instance, Ref.[3] for reviews), in which the reason of this phenomenon is due to an exotic component with large negative pressure, or modified theories of gravity[4]. Unfortunately, up to now a satisfactory answer to the question that what dark energy is and where it came from has not yet to be obtained. Alternative to dark energy, modified theories of gravity is extremely attractive, such as f(R) gravity (see, for instance, Ref.[5] for reviews), here f(R) is an arbitrary function of the Ricci scalar R. Cosmic acceleration can be explained by f(R) gravity[6], and the conditions of viable cosmological models have been derived in [7].

A general model of $f(R)$ gravity has been proposed in Ref.[8], which contains a non-minimal coupling between geometry and matter. This coupling term can be considered as a gravitational source to explain the current acceleration of the universe. As a result of the coupling the motion of the massive particles is non-geodesic, and an extra force, orthogonal to the four-velocity, arises. Different forms for the matter Lagrangian density L_m, and the resulting extra-force, were considered in [9], and it was shown that more natural forms for L_m do not imply the vanishing of the extra-force. The implications of the non-minimal coupling on the stellar equilibrium were investigated in [10], where

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constraints on the coupling were also obtained. An inequality which expresses a necessary and sufficient condition to avoid the Dolgov-Kawasaki instability for the model was derived in [11]. However, a more general model, in which the coupling style is arbitrary and the Lagrangian density of matter only appears in coupling term, has been proposed in Ref.[12], i.e., the so-called the generalized f(R) gravity with arbitrary coupling between matter and geometry. In this class of models the energy-momentum tensor of the matter is generally not conserved, and the matter-geometry coupling can induce a supplementary acceleration of the test particles. Moreover, the energy conditions and the Dolgov-Kawasaki criterion for the model have been derived in Ref.[13], which are quite general and can degenerate to the well-known energy conditions in GR and f(R) gravity with non-minimal coupling and non-coupling as special cases.

Another interesting alternative modified theory of gravity is the modified Gauss-Bonnet gravity, i.e., f(G) gravity, where f(G) is a general function of the Gauss-Bonnet (GB) term[14, 15]. At present specific models of f(G) gravity have been proposed to account for the late-time cosmic acceleration[15, 16], and the respective constraints on the parameters of the models have also analyzed in Ref.[16]. For more general forms of f(G), the most crucial condition to be satisfied is $d^2f/dG^2 > 0$, which is required to ensure the stability of a late-time de Sitter solution as well as the existence of standard radiation/matter dominated epochs[17], and solar system constraints on f(G) gravity models have been studied in Ref.[18]. Recently the energy conditions in f(G) gravity have been also discussed[19], but they are only adapted to $f(G)$ gravity without coupling between matter and geometry. Hence, in this paper the $f(G)$ gravity models with curvature-matter coupling will be proposed, and some relevant issues, such as the energy conditions, the stability criterion and the conditions for late-time cosmic accelerated expansion, will be studied.

This paper is organized as follows. In Section 2, the $f(G)$ gravity models with curvature-matter coupling are proposed, which is here called the modified $f(G)$ gravity. And some fundamental elements of the modified $f(G)$ gravity are given. In Section 3, the well-known energy conditions, namely, the strong energy condition (SEC), the null energy condition (NEC), the weak energy condition (WEC) and the dominant energy condition (DEC), will be derived. Concretely, the two models are applied to the weak energy condition in order to understand the meaning of these energy conditions. Furthermore, we will study stability criterion at the de Sitter point, by which the parameters in the specific model in the modified f(G) gravity can be constrained in Section 4. In addition, by using the conditions of power-law accelerated expansion and the equation of state of matter less than $-\frac{1}{3}$, the conditions for late-time cosmic accelerated expansion in the modified f(G) gravity are discussed in Section 5. Conclusions and discussions on our work are given in the last section.

II. FIELD EQUATIONS IN THE MODIFIED F(G) GRAVITY

As we know, the modified Einstein-Hilbert action[20] is,

$$S = \int d^4x\sqrt{-g}[\frac{R+f(G)}{2\kappa} + L_m], \tag{1}$$

in which $\kappa = 8\pi G_N$, G_N is the gravitational constant, $R = R(g_{\mu\nu})$ is the Ricci scalar, and L_m is the Lagrangian density of matter. The Gauss-Bonnet invariant is defined as $G \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\xi\sigma}R^{\mu\nu\xi\sigma}$ ($R_{\mu\nu}$ and $R_{\mu\nu\xi\sigma}$ are the Ricci tensor and the Riemann tensor, respectively).

Below, we consider f(G) gravity with curvature-matter coupling, which is here called the modified f(G) gravity. The

Lagrangian density of matter only appears in coupling term and the action is given by

$$S = \int d^4x\sqrt{-g}\{\frac{R}{2} + [1 + \lambda f(G)]L_m\}, \tag{2}$$

where we have chosen $\kappa = 8\pi G_N = 1$, which will be adopted hereafter. Varying the action (2) with respect to the metric $g^{\mu\nu}$ yields the field equations:

$$\begin{aligned} R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R \;=\; & T_{\mu\nu} + 2\lambda L_m(-2FRR_{\mu\nu} + 4FR^{\xi}_{\mu}R_{\nu\xi} - 2FR_{\mu\xi\sigma\varsigma}R^{\xi\sigma\varsigma}_{\nu} - 4FR_{\mu\xi\sigma\nu}R^{\xi\sigma} \\ & + 2R\nabla_\mu\nabla_\nu F - 2Rg_{\mu\nu}\nabla^2F - 4R^{\xi}_{\mu}\nabla_\nu\nabla_\xi F - 4R^{\xi}_{\nu}\nabla_\mu\nabla_\xi F \\ & + 4R_{\mu\nu}\nabla^2F + 4g_{\mu\nu}R^{\xi\sigma}\nabla_\xi\nabla_\sigma F - 4R_{\mu\xi\nu\sigma}\nabla^\xi\nabla^\sigma F), \end{aligned} \tag{3}$$

where $F = F(G) \equiv \partial f(G)/\partial G$, and $T_{\mu\nu}$ is the energy-momentum tensor of matter, which is defined as:

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \cdot \frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}}. \tag{4}$$

By assuming that the Lagrangian density L_m of the matter depends only on the metric tensor components, and not on its derivatives, we obtain $T_{\mu\nu} = L_m g_{\mu\nu} - 2\partial L_m/\partial g^{\mu\nu}$. By taking the covariant derivative of Eq.(3), with the use of the Bianchi identities, $\nabla^\mu G_{\mu\nu} = 0$ (here $G_{\mu\nu}$ is the Einstein tensor), we can obtain the following relation:

$$\begin{aligned} \nabla^\mu T_{\mu\nu} \;=\; & 4\lambda L_m[\nabla^\mu F(RR_{\mu\nu} - 2R^{\xi}_{\mu}R_{\nu\xi} + R_{\mu\xi\sigma\varsigma}R^{\xi\sigma\varsigma}_{\nu} + 2g^{\alpha\xi}g^{\beta\sigma}R_{\mu\alpha\nu\beta}R_{\xi\sigma}) \\ & + F(R_{\mu\nu}\nabla^\mu R + \tfrac{1}{2}g_{\mu\nu}R\nabla^\mu R - 2R_{\nu\xi}\nabla^\mu R^{\xi}_{\mu} - 2R^{\xi}_{\mu}\nabla^\mu R_{\nu\xi} + R^{\xi\sigma\varsigma}_{\nu}\nabla^\mu R_{\mu\xi\sigma\varsigma} \\ & + R_{\mu\xi\sigma\varsigma}\nabla^\mu R^{\xi\sigma\varsigma}_{\nu} + 2g^{\alpha\xi}g^{\beta\sigma}R_{\xi\sigma}\nabla^\mu R_{\mu\alpha\nu\beta} + 2g^{\alpha\xi}g^{\beta\sigma}R_{\mu\alpha\nu\beta}\nabla^\mu R_{\xi\sigma}) + R\nabla_\nu\Box F \\ & - R\Box\nabla_\nu F - \nabla_\mu\nabla_\nu F\nabla^\mu R - 2R_{\mu\nu}\nabla^\mu\Box F + 2R^{\xi}_{\mu}\nabla^\mu\nabla_\nu\nabla_\xi F + 2\nabla_\nu\nabla_\xi F\nabla^\mu R^{\xi}_{\mu} \\ & + 2R^{\xi}_{\nu}\Box\nabla_\xi F + 2\nabla_\mu\nabla_\xi F\nabla^\mu R^{\xi}_{\nu} - 2g_{\mu\nu}R^{\xi\sigma}\nabla^\mu\nabla_\xi\nabla_\sigma F - 2g_{\mu\nu}\nabla_\xi\nabla_\sigma F\nabla^\mu R^{\xi\sigma} \\ & + 2g^{\alpha\xi}g^{\beta\sigma}R_{\mu\alpha\nu\beta}\nabla^\mu\nabla_\xi\nabla_\sigma F + 2g^{\alpha\xi}g^{\beta\sigma}\nabla_\xi\nabla_\sigma F\nabla^\mu R_{\mu\alpha\nu\beta}], \end{aligned} \tag{5}$$

from which we see that the conservation of the energy-momentum tensor of matter is violated due to the coupling between matter and geometry.

For the perfect fluid, the energy-momentum tensor is

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}, \tag{6}$$

where u_μ is the four-velocity, and satisfies the conditions $u_\mu u^\mu = 1$ and $u^\mu u_{\mu;\nu} = 0$[8]. Thus, the covariant derivative of Eq.(6) can be given as

$$\nabla^\mu T_{\mu\nu} = (\rho + p)g_{\mu\lambda}u^\nu\nabla_\nu u^\mu - \nabla_\nu p\ (\delta^{\nu}_{\lambda} - u^\nu u_\lambda). \tag{7}$$

By imposing the condition of the conservation of the matter current, $\nabla_\nu(\rho u^\nu) = 0$, and with the use of the identity $u^\nu\nabla_\nu u^\mu = \frac{d^2x^\mu}{ds^2} + \Gamma^{\mu}_{\nu\lambda}u^\nu u^\lambda$[12], we have the equation of motion of a test particle in the model as

$$\frac{Du^\mu}{ds} \equiv \frac{du^\mu}{ds} + \Gamma^{\mu}_{\nu\lambda}u^\nu u^\lambda = \frac{d^2x^\mu}{ds^2} + \Gamma^{\mu}_{\nu\lambda}u^\nu u^\lambda = f^\mu, \tag{8}$$

where the extra-force f^μ has the following expression

$$f^\mu = \frac{1}{\rho + p}[\nabla^\mu T_{\mu\nu}g^{\mu\lambda} + \nabla_\nu p\ (g^{\mu\nu} - u^\nu u^\mu)]. \tag{9}$$

By substituting the relation (5) into Eq.(9), we can point that due to the presence of the coupling between matter and geometry, the motion of the massive particles is non-geodesic, and the extra-force f^μ is not orthogonal to the four-velocity u_μ, that is, there exists an angle between the extra-force f^μ and the four-velocity u_μ, and $f^\mu u_\mu \neq 0$.

III. ENERGY CONDITIONS IN THE MODIFIED F(G) GRAVITY

A. The Raychaudhuri Equation

The energy conditions arise when one refers to the Raychaudhuri equation for the expansion[21]. Under these energy conditions, one allows not only to establish gravity which remains attractive, but also to keep the demands that the energy density is positive and cannot flow faster than light. Below, following Ref.[13] we simply review the Raychaudhuri equation which is the physical origin of the null energy condition(NEC) and the strong energy condition(SEC)[22].

In the case of a congruence of timelike geodesics defined by the vector field u^{μ}, the Raychaudhuri equation is given by

$$\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}u^{\mu}u^{\nu}, \tag{10}$$

where $R_{\mu\nu}, \theta, \sigma_{\mu\nu}$ and $\omega_{\mu\nu}$ are the Ricci tensor, the expansion parameter, the shear and the rotation associated with the congruence, respectively. While in the case of a congruence of null geodesics defined by the vector field k^{μ}, the Raychaudhuri equation is given by

$$\frac{d\theta}{d\tau} = -\frac{1}{2}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}k^{\mu}k^{\nu}. \tag{11}$$

From above expressions, it is clear that the Raychaudhuri equation is a purely geometric statement and independent of the gravity theory. In order to constrain the energy-momentum tensor by the Raychaudhuri equation, one can use the Ricci tensor from the field equations of gravity to make a connection. Namely, through the combination of the field equations of gravity and the Raychaudhuri equation, one can obtain physical conditions for the energy-momentum tensor. Since $\sigma^2 \equiv \sigma_{\mu\nu}\sigma^{\mu\nu} \geq 0$ (the shear is a spatial tensor) and $\omega_{\mu\nu} = 0$ (hypersurface orthogonal congruence), from Eqs. (10) and (11), the conditions for gravity to remain attractive ($\frac{d\theta}{d\tau} < 0$) are

$$R_{\mu\nu}u^{\mu}u^{\nu} \geq 0 \qquad SEC, \tag{12}$$

$$R_{\mu\nu}k^{\mu}k^{\nu} \geq 0 \qquad NEC. \tag{13}$$

Thus by means of the relationship (12) and Einstein's equation ($R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T_{\mu\nu}$), one obtains

$$R_{\mu\nu}u^{\mu}u^{\nu} = (T_{\mu\nu} - \frac{T}{2}g_{\mu\nu})u^{\mu}u^{\nu} \geq 0. \tag{14}$$

If one considers a perfect fluid with energy density ρ and pressure p,

$$T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} + pg_{\mu\nu} \tag{15}$$

the relationship (14) turns into the well-known SEC in general relativity, i.e.,

$$\rho + 3p \geq 0. \tag{16}$$

Similarly, by using the relationship (13) and Einstein's equation, one has

$$T_{\mu\nu}k^{\mu}k^{\nu} \geq 0. \tag{17}$$

Thus, by considering Eq.(15), the familiar NEC in general relativity can be reproduced as:

$$\rho + p \geq 0. \tag{18}$$

B. Energy conditions

The Einstein tensor resulting from the field equation (3) can be written

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T^{eff}_{\mu\nu}, \tag{19}$$

where

$$\begin{aligned} T^{eff}_{\mu\nu} \;=\; & T_{\mu\nu} + 2\lambda L_m(-2FRR_{\mu\nu} + 4FR^{\xi}_{\mu}R_{\nu\xi} - 2FR_{\mu\xi\sigma\varsigma}R^{\xi\sigma\varsigma}_{\nu} - 4FR_{\mu\xi\sigma\nu}R^{\xi\sigma} \\ & +2R\nabla_\mu\nabla_\nu F - 2Rg_{\mu\nu}\nabla^2 F - 4R^{\xi}_{\mu}\nabla_\nu\nabla_\xi F - 4R^{\xi}_{\nu}\nabla_\mu\nabla_\xi F \\ & +4R_{\mu\nu}\nabla^2 F + 4g_{\mu\nu}R^{\xi\sigma}\nabla_\xi\nabla_\sigma F - 4R_{\mu\xi\nu\sigma}\nabla^\xi\nabla^\sigma F). \end{aligned} \tag{20}$$

Contracting the above equation, we have

$$T^{eff} = T - 4\lambda L_m(FG + R\Box F - 2R_{\mu\nu}\nabla^\mu\nabla^\nu F). \tag{21}$$

Thus, we can write $R_{\mu\nu}$ in terms of an effective stress-energy tensor and its trace, i.e. ,

$$R_{\mu\nu} = T^{eff}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T^{eff}. \tag{22}$$

By using the relationship (12) and Eq.(22), the SEC can be given as:

$$T^{eff}_{\mu\nu}u^\mu u^\nu - \frac{1}{2}T^{eff} \geq 0. \tag{23}$$

By using the Eqs.(20) and (21), the SEC in Eq.(23) can be expressed as

$$\rho + 3p + 8\lambda L_m FG - 96\lambda L_m H^3 \dot{f_{,G}} + 4\lambda L_m R\Box F - 8\lambda L_m R_{\mu\nu}\nabla^\mu\nabla^\nu F \geq 0, \tag{24}$$

where $H = \dot{a}(t)/a(t)$ is the Hubble parameter. The NEC in the modified f(G) gravity can be expressed as:

$$T^{eff}_{\mu\nu}k^\mu k^\nu \geq 0. \tag{25}$$

By the same method as the SEC, the NEC in Eq.(25) can be changed into

$$\rho + p + 4\lambda L_m FG - 64\lambda L_m H^3 \dot{f_{,G}} + \frac{4}{3}\lambda L_m R\Box F - \frac{8}{3}\lambda L_m R_{\mu\nu}\nabla^\mu\nabla^\nu F \geq 0. \tag{26}$$

Furthermore, by means of Eqs.(20) and (22), the effective energy density and the effective pressure can be derived as follows:

$$\rho^{eff} = 2\lambda L_m(FG - 24H^3\dot{f_{,G}}) + \rho, \tag{27}$$

$$p^{eff} = 2\lambda L_m FG - 16\lambda L_m H^3\dot{f_{,G}} + \frac{4}{3}\lambda L_m R\Box F - \frac{8}{3}\lambda L_m R_{\mu\nu}\nabla^\mu\nabla^\nu F + p. \tag{28}$$

Note that the above expressions of the SEC (24) and the NEC (26) are directly derived from Raychaudhuri equation. However, equivalent results can obtained by the transformations $\rho \to \rho^{eff}$ and $p \to p^{eff}$ into $\rho + 3p \geq 0$ and $\rho + p \geq 0$. Thus by extending this approach to $\rho - p \geq 0$ and $\rho \geq 0$, the corresponding DEC and the WEC in the modified f(G) gravity can be respectively given as:

$$\rho - p - 32\lambda L_m H^3\dot{f_{,G}} - \frac{4}{3}\lambda L_m R\Box F + \frac{8}{3}\lambda L_m R_{\mu\nu}\nabla^\mu\nabla^\nu F \geq 0, \tag{29}$$

$$\rho+2\lambda L_m FG-48H^3\dot{f_{,G}}\geq 0. \tag{30}$$

It is worth stressing that when taking $f(G)=0$, the energy conditions in general relativity can be reproduced.

Furthermore, by defining the deceleration, jerk, and snap parameters as[23]

$$q=-\frac{1}{H^2}\cdot\frac{\ddot{a}}{a},\ j=\frac{1}{H^3}\cdot\frac{\dddot{a}}{a},\ s=\frac{1}{H^4}\cdot\frac{\ddddot{a}}{a}, \tag{31}$$

we have

$$\dot{H}=-H^2(1+q),\ddot{H}=H^3(j+3q+2),\dddot{H}=H^4(s-2j-5q-3), \tag{32}$$

and the GB term is given by

$$G=24H^2(H^2+\dot{H}). \tag{33}$$

In addition, we consider $L_m=-\rho$, we can rewrite Eqs. (27) and (28) as follows:

$$\rho^{eff}=\rho+48\lambda H^4\rho[qf'+24H^4(2q^2+3q+j)f''], \tag{34}$$

$$p^{eff}=p+48\lambda H^4\rho[qf'+4H^4(-2q^3-8q^2+q-2j-6qj+s+3)f''+96H^8(2q^2+3q+j)^2f''']. \tag{35}$$

Hence, the energy conditions, i.e., the SEC, NEC, DEC and WEC can be rewritten as:

$$\begin{aligned}\rho+3p+48\lambda H^4\rho[qf'+24H^4(2q^2+3q+j)f'']+144\lambda H^4\rho[qf'+4H^4(-2q^3\\ -8q^2+q-2j-6qj+s+3)f''+96H^8(2q^2+3q+j)^2f''']\geq 0,\end{aligned} \tag{36}$$

$$\begin{aligned}\rho+p+48\lambda H^4\rho[qf'+24H^4(2q^2+3q+j)f'']+48\lambda H^4\rho[qf'+4H^4(-2q^3\\ -8q^2+q-2j-6qj+s+3)f''+96H^8(2q^2+3q+j)^2f''']\geq 0,\end{aligned} \tag{37}$$

$$\begin{aligned}\rho-p+48\lambda H^4\rho[qf'+24H^4(2q^2+3q+j)f'']-48\lambda H^4\rho[qf'+4H^4(-2q^3\\ -8q^2+q-2j-6qj+s+3)f''+96H^8(2q^2+3q+j)^2f''']\geq 0,\end{aligned} \tag{38}$$

$$\rho+48\lambda H^4\rho[qf'+24H^4(2q^2+3q+j)f'']\geq 0. \tag{39}$$

It is worth stressing that when taking $L_m=p$, Eqs.(27) and (28) change into

$$\rho^{eff}=\rho-48\lambda H^4p[qf'+24H^4(2q^2+3q+j)f''], \tag{40}$$

$$p^{eff}=p-48\lambda H^4p[qf'+4H^4(-2q^3-8q^2+q-2j-6qj+s+3)f''+96H^8(2q^2+3q+j)^2f''']. \tag{41}$$

And the corresponding energy conditions are as follows:

$$\begin{aligned}\rho+3p-48\lambda H^4p[qf'+24H^4(2q^2+3q+j)f'']-144\lambda H^4p[qf'+4H^4(-2q^3\\ -8q^2+q-2j-6qj+s+3)f''+96H^8(2q^2+3q+j)^2f''']\geq 0,\end{aligned} \tag{42}$$

$$\rho + p - 48\lambda H^4 p[qf' + 24H^4(2q^2 + 3q + j)f''] - 48\lambda H^4 p[qf' + 4H^4(-2q^3 -8q^2 + q - 2j - 6qj + s + 3)f'' + 96H^8(2q^2 + 3q + j)^2 f'''] \geq 0, \tag{43}$$

$$\rho - p - 48\lambda H^4 p[qf' + 24H^4(2q^2 + 3q + j)f''] + 48\lambda H^4 p[qf' + 4H^4(-2q^3 -8q^2 + q - 2j - 6qj + s + 3)f'' + 96H^8(2q^2 + 3q + j)^2 f'''] \geq 0, \tag{44}$$

$$\rho - 48\lambda H^4 p[qf' + 24H^4(2q^2 + 3q + j)f''] \geq 0. \tag{45}$$

C. Energy Conditions for specific f(G) models

In order to exemplify how to use the energy conditions to constrain the f(G) theories of gravity, below, we study the realistic models of f(G) gravity, which have been found to reproduce the current acceleration[15, 16]:

$$f_1(G) = \frac{a_1 G^n + b_1}{a_2 G^n + b_2}, \tag{46}$$

$$f_2(G) = a_3 G^n (1 + b_3 G^m), \tag{47}$$

where $a_1, a_2, a_3, b_1, b_2, b_3, n, m$ are all constants. Since there has been no reliable measurement for the snap parameter (s) up to now, we only focus on the WEC (39) and (45) in this particular case.

Since the inequalities are too complicated to find exact analytical expressions, so we have to take some specific values of the parameters, such as $a_1 = b_1 = -1, a_2 = 2, b_2 = b$[19]. Also, when $G \to \pm\infty$ or $G \to 0^-$, the model (47) can be changed in the form $f(G) \sim \alpha G^n$[16], which means $a_3 = \alpha, b_3 = 0$. The above two models can be rewritten as:

$$f_1(G) = -\frac{G^n + 1}{2G^n + b}, \tag{48}$$

$$f_2(G) = \alpha G^n. \tag{49}$$

(1) When $L_m = -\rho$, we can get the corresponding WEC as follows:

$$\rho\{1 + 48\lambda q H^4 \frac{G^{n-1}(-nb + 2n)}{(2G^n + b)^2} + 1152\lambda H^8(2q^2 + 3q + j) \times \frac{G^{n-2} n[2G^n(n+1) - (n-1)b](-2 + b)}{(2G^n + b)^3}\} \geq 0 \quad (for f_1(G)), \tag{50}$$

$$\rho[1 + 48\lambda q H^4 \alpha n G^{n-1} + 1152\lambda H^8(2q^2 + 3q + j)\alpha n(n-1)G^{n-2}] \geq 0 \quad (for f_2(G)). \tag{51}$$

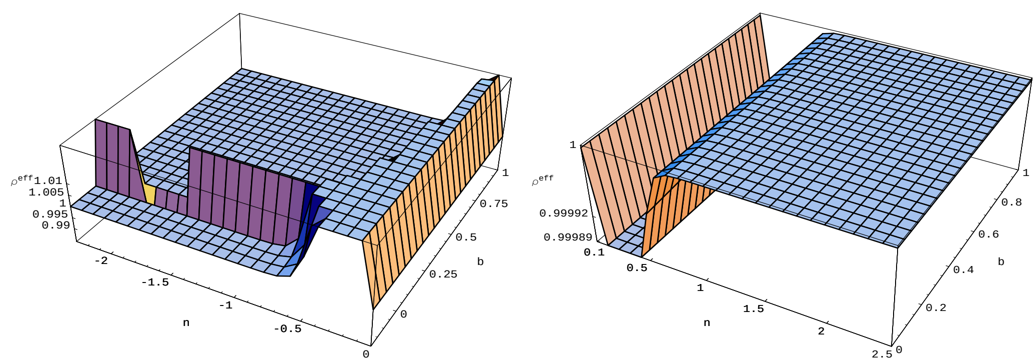
After a series of simplification, taking some present values $H_0 = 70.5$[24], $q_0 = -0.81 \pm 0.14$ and $j_0 = 2.16^{+0.81}_{-0.75}$[25], and $\lambda = 1$, we can obtain the restrictions on the parameters n, b and α, which satisfy the WEC in Eqs.(50) and (51), respectively (see Figs.1 and 2). Fig.1 shows that the WEC $\rho^{eff} > 0$ can give the constraints on the parameters n and b in the $f_1(G)$ model, i.e., $0 \leq b \leq 1$ and $-2.3 \leq n \leq 2.5$, but except $|n| < 0.1$ owing to the non-continuity of $\rho^{eff} > 0$. Similarly, Fig.2 illustrates that only when $\alpha > 0$ and $7.5 \lesssim n \lesssim 10$, the WEC $\rho^{eff} > 0$ can be satisfied in the $f_2(G)$ model.

(2) When $L_m = p$, let us consider a perfect fluid composed of non-relativistic or relativistic particles with constant barotropic equation of state (EoS) $p = (\gamma - 1)\rho$, $0 \leq \gamma \leq 2$ is a constant relating to the EoS by $\omega = \gamma - 1$[26–28], the corresponding WEC changes into:

$$\rho\{1 - 48\lambda q\omega H^4 \frac{G^{n-1}(-nb+2n)}{(2G^n+b)^2} - 1152\lambda H^8\omega(2q^2+3q+j) \\ \times \frac{G^{n-2}n[2G^n(n+1)-(n-1)b](-2+b)}{(2G^n+b)^3}\} \geq 0 \quad (for f_1(G)), \tag{52}$$

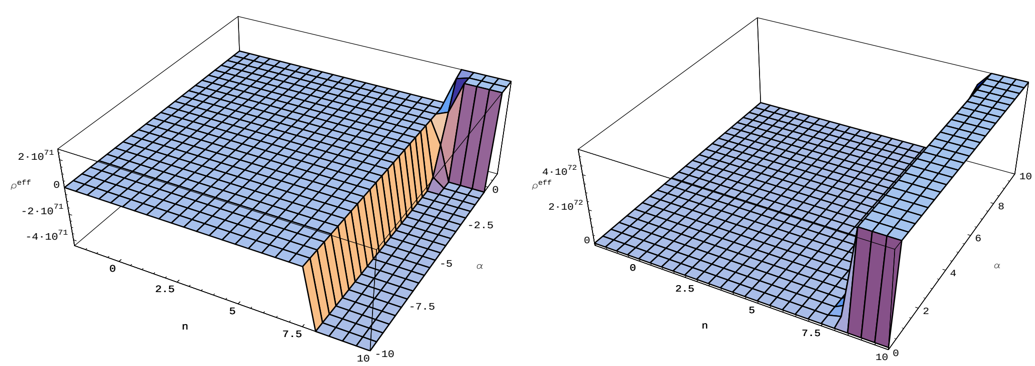
$$\rho[1 - 48\lambda q\omega H^4 \alpha n G^{n-1} - 1152\lambda H^8\omega(2q^2+3q+j)\alpha n(n-1)G^{n-2}] \geq 0 \quad (for f_2(G)). \tag{53}$$

By the similar discussions to the case of $L_m = -\rho$, we can obtain the restrictions on the parameters n, b and α, which respectively satisfy the WEC in Eqs.(52) and (53), and are illustrated in Figs.3 and 4 (here taking $\omega = 0.5$). From Fig.3 it is easy to see that the constraints on the parameters n and b for the $f_1(G)$ model are $-0.2 \leq b \leq 1$ and $-2.3 \leq n \leq 2.5$, which are nearly the same as the results of the $L_m = -\rho$, but in the Fig.4 $-10 \leq \alpha \leq 10$ and $-2.7 \leq n \leq -1.3$ for the $f_2(G)$ model are quite different from the results of the $L_m = -\rho$.



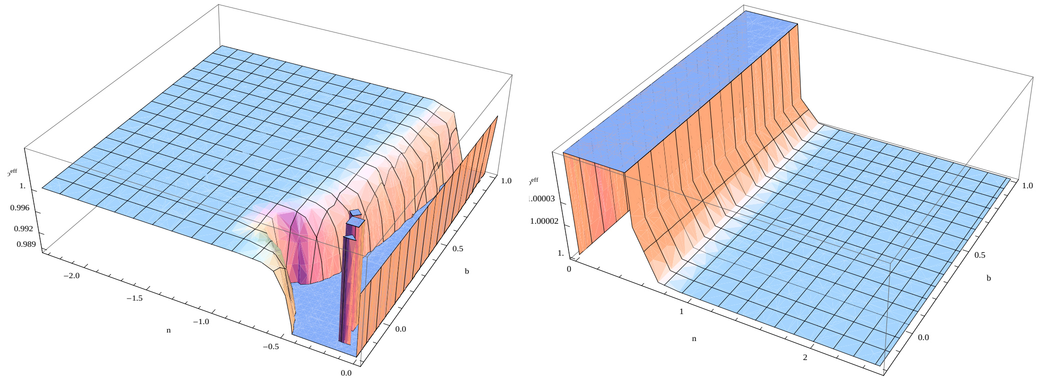
(a) (b)

FIG. 1: The constraints of WEC ($\rho^{eff} > 0$) on the parameters n and b for the $f_1(G)$ model in (48) with $\lambda = 1$ and $L_m = -\rho$.



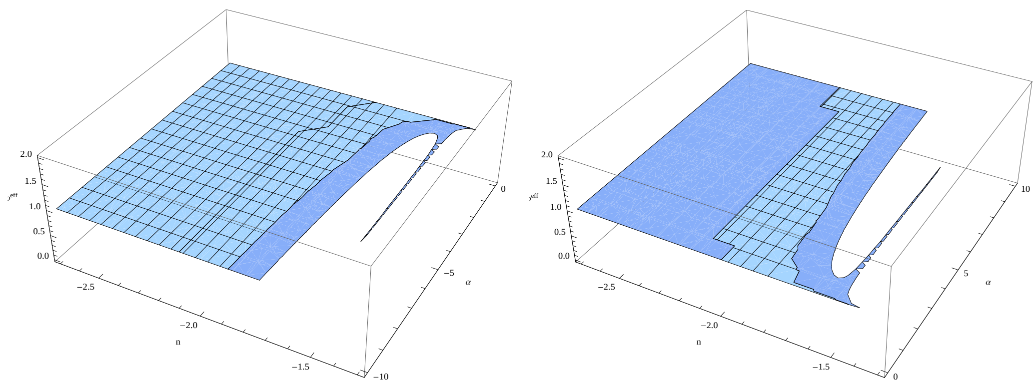
(a) (b)

FIG. 2: The constraints of WEC ($\rho^{eff} > 0$) on the parameters n and α for the $f_2(G)$ model in (49) with $\lambda = 1$ and $L_m = -\rho$.



(a) (b)

FIG. 3: The constraints of WEC ($\rho^{eff} > 0$) on the parameters n and α for the $f_1(G)$ model in (48) with $\lambda = 1$ and $L_m = p = \omega\rho$ ($\omega = 0.5$).



(a) (b)

FIG. 4: The constraints of WEC ($\rho^{eff} > 0$) on the parameters n and α for the $f_2(G)$ model in (49) with $\lambda = 1$ and $L_m = p = \omega\rho$ ($\omega = 0.5$).

IV. STABILITY CRITERION AT DE SITTER POINT

Modified gravity must be stable at the classical and quantum level. There are in principle several kinds of instabilities to consider, such as Dolgov-Kawasaki criterion in f(R) gravity[29]. Below, following Ref.[17], we will focus on the stability criterion at de Sitter point in the modified f(G) gravity.

In a flat FLRW background with the metric

$$ds^2 = -dt^2 + a(t)^2 dX_3^2, \tag{54}$$

where $a(t)$ is the scale factor and dX_3^2 contains the spacial part of the metric. The 00 component of the field equation (3) gives

$$3H^2 = 2\lambda L_m G f_{,G} - 48\lambda L_m H^3 \dot{f_{,G}} + \rho_m + \rho_r. \tag{55}$$

Let us first discuss the stability around the de Sitter point in the modified f(G) gravity by neglecting the contribution of pressure-less matter ρ_m and radiation ρ_r. The Hubble parameter, $H = H_1$ (at the de Sitter point), satisfies

$$3H_1^2 = 2\lambda L_m G_1 f_{,G}(G_1) - 48\lambda L_m H_1^3 \dot{f_{,G}}(G_1), \tag{56}$$

where $G_1 = 24H_1^4$, and the relations $\dot{H}_1 = 0$ and $\dot{G}_1 = 0$ are used. Considering a linear perturbation δH_1 about the de Sitter point, Eq.(55) gives

$$2\lambda L_m(24H_1^3 f_{,GG}\delta\dot{G}_1 - f_{,G}\delta G_1) = \delta H_1(48\lambda L_m H_1^4 f_{,GG} - 6H_1). \tag{57}$$

Substituting the relations $\delta G(H_1) = 24(4H_1^3\delta H_1 + H_1^2\delta\dot{H}_1)$ and $\delta\dot{G}(H_1) = 24H_1^2(\delta\ddot{H}_1 + 4H_1\delta\dot{H}_1)$ into Eq.(57), we obtain

$$\delta\ddot{H}_1 + \frac{96H_1^4 f_{,GG} - f_{,G}}{24H_1^3 f_{,GG}}\delta\dot{H}_1 + (\frac{1}{192\lambda L_m H_1^4 f_{,GG}} - \frac{1}{24H_1} - \frac{f_{,G}}{6H_1^2 f_{,GG}})\delta H_1 = 0. \tag{58}$$

It follows that the effective mass squared is $(\frac{1}{192\lambda L_m H_1^4 f_{,GG}} - \frac{1}{24H_1} - \frac{f_{,G}}{6H_1^2 f_{,GG}})$, which must be non-negative for stability. Therefore, we can obtain

$$\frac{1 - 8\lambda H_1^3 f_{,GG} - 32\lambda L_m H_1^2 f_{,G}}{192\lambda L_m H_1^4 f_{,GG}} > 0, \tag{59}$$

which is just the stability criterion at the de Sitter point. It follows that if the exact value of H_1 and a suitable form of L_m can be given in the modified f(G) gravity models, then the constraints on the parameters in the specific model can be obtained.

V. THE CONDITIONS FOR LATE-TIME COSMIC ACCELERATED EXPANSION IN THE MODIFIED F(G) GRAVITY

It is known that late-time cosmic accelerated expansion occurs under the conditions of either a power-law expansion or the equation of state of matter less than $-\frac{1}{3}$. To exemplify how to use these conditions to realize the phase of accelerating expansion in the modified f(G) gravity, now we concentrate on the model $f_1(G)$ in (48). Thus, by means of the action (2) and the energy density L_m of perfect fluid[30, 31], i.e.,

$$L_m = -\rho = -\rho_0 a^{-3(1+\omega)}, \tag{60}$$

where ω is the equation of state of perfect fluid and is taken to be a constant, the field equation (55) becomes

$$3H^2 = -\rho_0 a^{-3(1+\omega)}[1 - 48\lambda H^2(H^2 + \dot{H})f' - 1152H^4(2\dot{H}^2 + H\ddot{H} + 4H^2\dot{H})f'']. \tag{61}$$

Note that the relation $R = 6(2H^2 + \dot{H})$ is used in the derivation of the Eq.(61).

If assuming the solution of (61) is $a = a_0 t^r$[13], then we have $H = \frac{r}{t}$, $\dot{H} = -\frac{r}{t^2}$, $\ddot{H} = \frac{2r}{t^3}$. Substituting all these relations into (61), we can get the following equation:

$$\begin{aligned}
3r^2 \;=\; & \rho_0 a_0^{-3(1+\omega)} t^{-3r(1+\omega)+2} \frac{1}{(r-1)[b + 2^{3n+1}\times 3^n \frac{(r-1)^n r^{3n}}{t^{4n}}]^3} \times \{b^3(r-1) \\
& + 8\frac{(r-1)^{2n} r^{6n}}{t^{8n}}[-13824^n \times \frac{(r-1)^n r^{3n}}{t^{4n}} + 13824^n \times \frac{r(r-1)^n r^{3n}}{t^{4n}} + 5576^n n\lambda \\
& + 4^{3n+1}\times 9^n n^2\lambda - 576^n nr\lambda] + 2^{3n+1}\times 3^n b^2 \frac{(r-1)^n r^{3n}}{t^{4n}}[-3 - 5n\lambda + 4n^2\lambda \\
& + r(3 + n\lambda)] + 4b\frac{(r-1)^n r^{3n}}{t^{4n}}\{-3^{2n+1}\times 64^n \frac{(r-1)^n r^{3n}}{t^{4n}} + 5n\lambda[24^n \\
& - 576^n \frac{(r-1)^n r^{3n}}{t^{4n}}] - 4n^2\lambda[24^n + 2^{6n+1}\times 9^n \frac{(r-1)^n r^{3n}}{t^{4n}} - 576^n \frac{(r-1)^n r^{3n}}{t^{4n}}] \\
& + r[3^{2n+1}\times 64^n \frac{(r-1)^n r^{3n}}{t^{4n}} - 24^n n\lambda + 576^n n\lambda \frac{(r-1)^n r^{3n}}{t^{4n}}]\}\}.
\end{aligned} \tag{62}$$

We find six kinds of possible relationships among r, ω and n, namely, $r = \frac{2(1-2n)}{3(1+\omega)}$, $r = \frac{2(1-4n)}{3(1+\omega)}$, $r = \frac{2(1-6n)}{3(1+\omega)}$, $r = \frac{2(1-8n)}{3(1+\omega)}$, $r = \frac{2(1-10n)}{3(1+\omega)}$ and $r = \frac{2(1-12n)}{3(1+\omega)}$. Under the condition of a power-law expansion (i.e., $r > 1$), the corresponding regions of ω are $\omega < -\frac{1}{3} - \frac{4}{3}n$ for $r = \frac{2(1-2n)}{3(1+\omega)}$, $\omega < -\frac{1}{3} - \frac{8}{3}n$ for $r = \frac{2(1-4n)}{3(1+\omega)}$, $\omega < -\frac{1}{3} - 4n$ for $r = \frac{2(1-6n)}{3(1+\omega)}$, $\omega < -\frac{1}{3} - \frac{16}{3}n$ for $r = \frac{2(1-8n)}{3(1+\omega)}$, $\omega < -\frac{1}{3} - \frac{20}{3}n$ for $r = \frac{2(1-10n)}{3(1+\omega)}$ and $\omega < -\frac{1}{3} - 8n$ for $r = \frac{2(1-12n)}{3(1+\omega)}$, respectively. Furthermore, by considering the equation of state of matter less than $-\frac{1}{3}$ (i.e., $\omega < -\frac{1}{3}$), the relationship among r, ω and n, condition and candidate for late-time cosmic accelerated expansion are shown in Table 1. The candidate for late-time cosmic accelerated expansion can be either the effective quintessence ($-1 < \omega < -\frac{1}{3}$) or the effective phantom ($\omega < -1$).

Relationship	Condition$(r > 1, \omega < -\frac{1}{3})$	The effective quintessence	The effective phantom
$r = \frac{2(1-2n)}{3(1+\omega)}$	$n > 0$ and $n \neq \frac{1}{2}$	$0 < n < \frac{1}{2}$	$n > \frac{1}{2}$
$r = \frac{2(1-4n)}{3(1+\omega)}$	$n > 0$ and $n \neq \frac{1}{4}$	$0 < n < \frac{1}{4}$	$n > \frac{1}{4}$
$r = \frac{2(1-6n)}{3(1+\omega)}$	$n > 0$ and $n \neq \frac{1}{6}$	$0 < n < \frac{1}{6}$	$n > \frac{1}{6}$
$r = \frac{2(1-8n)}{3(1+\omega)}$	$n > 0$ and $n \neq \frac{1}{8}$	$0 < n < \frac{1}{8}$	$n > \frac{1}{8}$
$r = \frac{2(1-10n)}{3(1+\omega)}$	$n > 0$ and $n \neq \frac{1}{10}$	$0 < n < \frac{1}{10}$	$n > \frac{1}{10}$
$r = \frac{2(1-12n)}{3(1+\omega)}$	$n > 0$ and $n \neq \frac{1}{12}$	$0 < n < \frac{1}{12}$	$n > \frac{1}{12}$

TABLE I: The relationship among r, ω and n, condition and candidate for late-time cosmic accelerated expansion in case $f(G) = -\frac{G^n+1}{2G^n+b}$.

From the above discussions, it is easy to see that the results in the model are interesting. Compared with the f(R) models, f(G) models are even more complicated. Since the Hubble parameter can be expressed as $H = \frac{r}{t}$, the GB term turns into $G = \frac{24r^3(r-1)}{t^4}$. If $0 < r < 1$, the early universe is in deceleration phase, which corresponds to the matter dominated phase with $r = \frac{2}{3}$, and if $r > 1$, the late universe is in acceleration phase.

Note that for the case of $L_m = p = \omega\rho$ we can make similar discussions to ones in the case of $L_m = -\rho$, and obtain the same results as the ones shown in Table 1 due to the constant ω.

VI. CONCLUSIONS AND DISCUSSIONS

In the present paper we have considered a modified f(G) gravity model with coupling between matter and geometry, described by the product of the Lagrange density of the matter and an arbitrary function of the Gauss-Bonnet term. The proposed action represents the general extension of the standard Hilbert action for the gravitational field, $S = \int d^4x\sqrt{-g}\{\frac{R}{2}+[1+\lambda f(G)]L_m\}$. The field equations and the equations of motion corresponding to this model show the non-conservation of the energy-momentum tensor, the presence of an extra-force acting on test particles and the non-geodesic motion. Moreover, in the modified f(G) gravity we have derived the energy conditions (SEC, NEC, DEC, WEC) when we consider $L_m = -\rho$ and $L_m = p$, respectively. For the SEC and the NEC, the Raychaudhuri equation, which is the physical origin of them, has been used. From the derivation, we found equivalent results can be obtained by taking the transformations $\rho \to \rho^{eff}$ and $p \to p^{eff}$ into $\rho + 3p \geq 0$ and $\rho + p \geq 0$. By means of these transformations, the DEC and WEC in the modified f(G) gravity have been also obtained. In order to exemplify how to use these energy conditions to constrain the modified f(G) gravity models, we have considered two specific models of f(G) gravity, i.e., $f_1(G)$ and $f_2(G)$ and given the corresponding constraints on the parameters in the $f_1(G)$ and

$f_2(G)$ models. Since there has been no reliable measurement for the snap parameter (s) up to now, we only focus on the WEC in this particular case. By analysis on Figs.1 and 2 we have given the constraints on the parameters in the $f_1(G)$ and $f_2(G)$ models satisfying the weak energy conditions when $L_m = -\rho$. By the similar discussions to the case of $L_m = -\rho$, when $L_m = p$ the restrictions on the parameters n, b and α have been also illustrated in Figs.3 and 4, from which we have found that in the two different forms of L_m the constraints on the parameters for the $f_1(G)$ model are nearly the same, but quite different for the $f_2(G)$ model. Furthermore, we have derived the stability criterion at the de Sitter point for the modified f(G) gravity models, which means that the modified f(G) gravity models may be stable. In addition, we have researched the conditions for late-time cosmic accelerated expansion in the modified f(G) gravity. Concretely, for the two different forms of L_m, the relationship among r, ω and n have been respectively given in the model $f(G) = -\frac{G^n+1}{2G^n+b}$, and by using the conditions of power-law accelerated expansion and the equation of state of matter less than $-\frac{1}{3}$, the constraints on the parameter n have been obtained, which are exactly the same in the two different forms of L_m. The candidate for late-time cosmic accelerated expansion would be either the effective quintessence ($-1 < \omega < -\frac{1}{3}$) or the effective phantom ($\omega < -1$), which could be determined by choosing n properly. Of course, other forms of f(G) gravity models with curvature-matter coupling will be considered in our following investigations.

Acknowledgments The research work is supported by the National Natural Science Foundation of China (11147150), the Natural Science Foundation of Education Department of Liaoning Province (L2011189), the Natural Science Foundation of Liaoning Province, China (Grant No.20102124) the NSFC (11175077) of P.R. China.

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